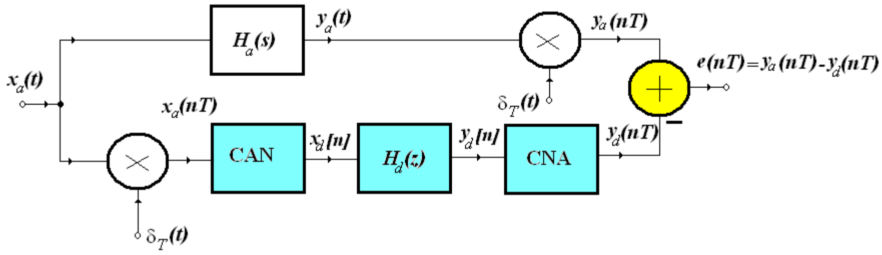
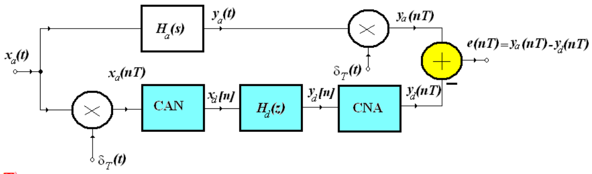
Transformarea sistemelor in timp continuu in sisteme in timp discret

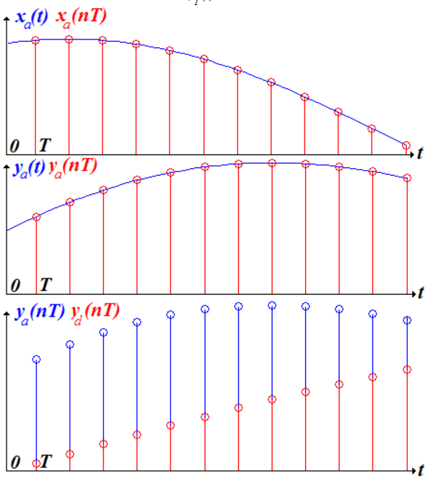
O data cu dezvoltarea tehnicii de calcul se pune tot mai fecvent problema inlocuirii sistemelor in timp continuu cu sisteme in timp discret, chiar si in aplicatiile semnalelor analogice.

Datorita experientei acumulate in proiectarea sistemelor in timp continuu, sunt de interes metodele de sinteza a sistemelor in timp discret bazate pe echivalarea acestora cu sisteme in timp continuu corespunzatoare.

O schema generala de evaluarea calitatii unei transformari pentru sistemele de banda limitata

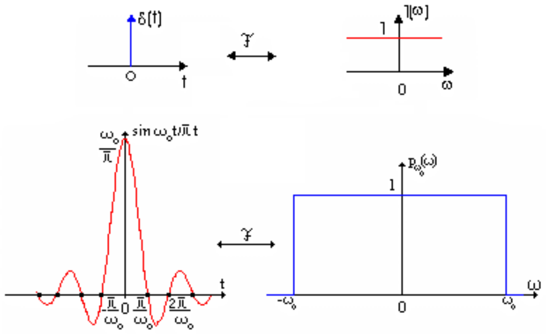






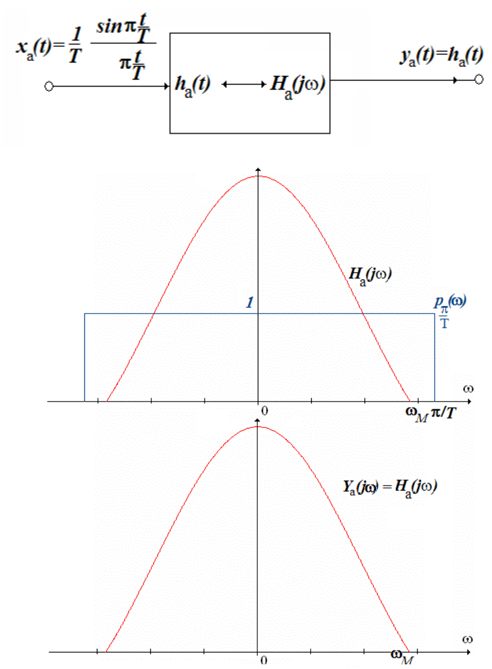
$H_a(s)$ - sistem de echivalat;
$H_d(z)$ - sistem echivalent.
Echivalarea trebuie facuta astfel incat eroarea e sa fie cat mai mica. Se presupune ca : $H_a(j\omega)=0$, ptr. $|\omega|>\omega_M$ si ca $|X_a(\omega)|=0$, ptr. $|\omega|>\omega_M$. Semnalul $x_a(t)$ nu poate fi identic cu $\delta(t)$ deoarece produsul $\delta(t)\cdot\delta_T(t)$ nu este definit.

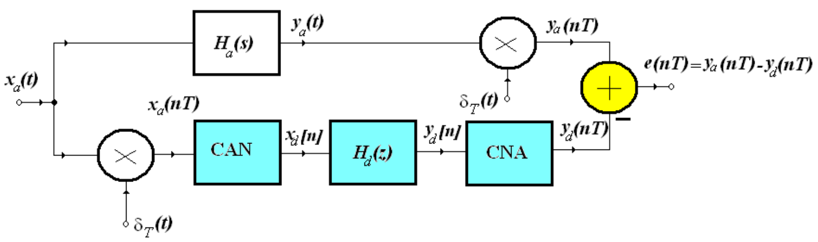
Semnalul de intrare

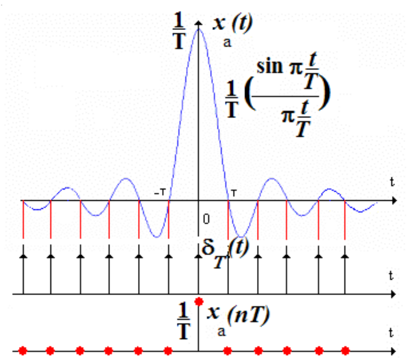


Semnalul $\delta(t)$ este folosit pentru identificarea raspunsului la impuls al unui sistem, nu neaparat de banda limitata. Semnalul $sin(\pi t)/\pi t$ este mai potrivit pentru estimarea raspunsului la impuls al unui sistem de banda limitata

cu $\omega_M < \dfrac{\pi}{T}$.







$y_a(t)=h_a(t)$; $x_d[n]=\frac{1}{T}\delta[n]$.

$y_d[n]=\frac{1}{T}\delta[n]*h_d[n]=\frac{1}{T}h_d[n]$;

$y_a(nT)=h_a(nT)$.

Pentru ca eroarea $e(nT)$ sa fie nula :

$h_a(nT)=\frac{1}{T}h_d(nT)\Rightarrow h_d[n]=Th_a(nT)$.

Fie $x_a(t) \leftrightarrow X_a(s)$, *un semnal de banda limitata la* π/T, *jumatate din frecventa de esantionare* $\omega_e, \omega_e = 2\pi/T$.

Raspunsul sistemului analogic, la momentul nT *este :*

$$y_a(nT) = \mathcal{L}^{-1}\{X_a(s)H_a(s)\}(nT).$$

Pe de alta parte :

$$x_d[n] = x_a(nT) \leftrightarrow X_d(z) \Rightarrow Y_d(z) = X_d(z)H_d(z) \Rightarrow$$

$$\Rightarrow y_d(nT) = y_d[n] = \mathcal{Z}^{-1}\{X_d(z)H_d(z)\}.$$

Din conditia de echivalare fara erori, $e(nT) = 0$, *rezulta :*

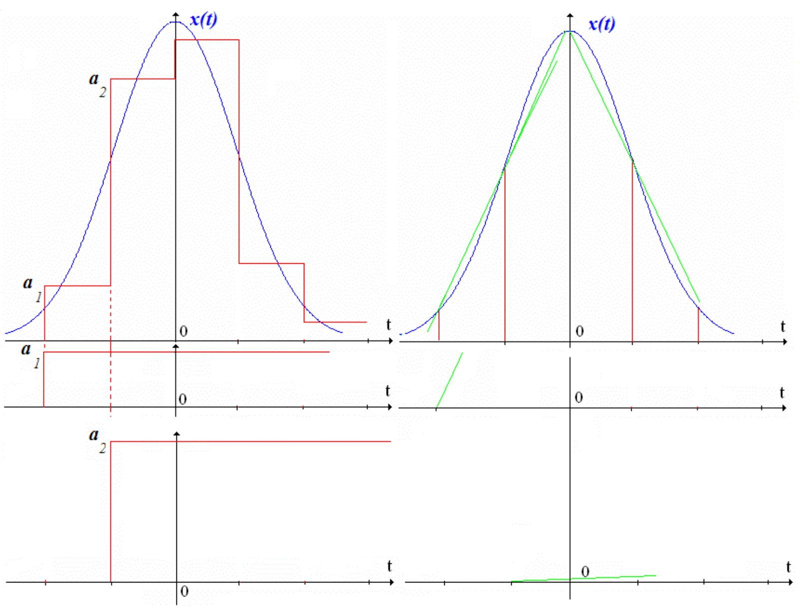
$$y_d[n] = y_d(nT) = y_a(nT), \text{ adica :}$$

$$\mathcal{Z}^{-1}\{X_d(z)H_d(z)\} = \mathcal{L}^{-1}\{X_a(s)H_a(s)\}(nT),$$

adica :

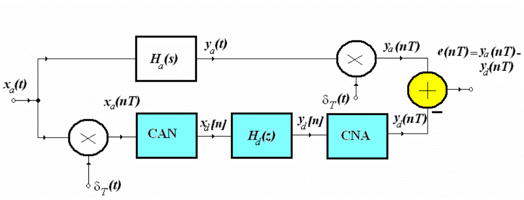
$$H_d(z) = \frac{1}{X_d(z)}\mathcal{Z}\left\{\mathcal{L}^{-1}\{X_a(s)H_a(s)\}(nT)\right\}$$

Se constata ca functia de transfer a sistemului numeric echivalent depinde de semnalul de intrare prin intermediul functiilor $X_a(s)$ *si* $X_d(z)$.



Nu exista nici un sistem digital perfect echivalent, oricarui sistem analogic, pentru orice semnal de intrare.

Echivalarea sistemelor de banda limitata prin metoda invariantei raspunsului la impuls



$$x_a(t)=\frac{1}{T}\cdot\frac{\sin\frac{\pi t}{T}}{\frac{\pi t}{T}}\;;\;x_d[n]=\frac{1}{T}\delta[n];\,h_d[n]=Th_a(nT);$$

$$\hat{Y}_a(s)=\mathcal{L}\{y_a(t)\delta_T(t)\}=\mathcal{L}\left\{\sum_{n=-\infty}^{\infty}h_a(nT)\delta(t-nT)\right\}=$$

$$=\sum_{n=-\infty}^{\infty}h_a(nT)\mathcal{L}\{\delta(t-nT)\}=\sum_{n=-\infty}^{\infty}h_a(nT)e^{-snT}\;;$$

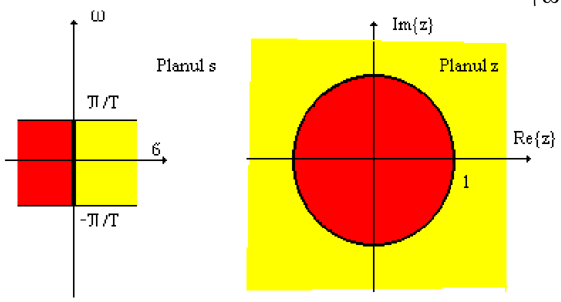
$$H_d(z)=\sum_{n=-\infty}^{\infty}h_d[n]z^{-n}=\sum_{n=-\infty}^{\infty}Th_a(nT)z^{-n}\;;\hat{Y}_a(s)=\frac{1}{T}H_d(z)\Big|_{z=e^{sT}}\;;$$

$$\hat{Y}_a(j\omega)=\frac{1}{T}\sum_{k=-\infty}^{\infty}H_a\left(j\omega-j\frac{2k\pi}{T}\right)\overset{j\omega=s}{\Leftrightarrow}\hat{Y}_a(s)=\frac{1}{T}\sum_{k=-\infty}^{\infty}H_a\left(s-j\frac{2k\pi}{T}\right).$$

$$H_d(z)\Big|_{z=e^{sT}}=\sum_{k=-\infty}^{\infty}H_a\left(s-j\frac{2k\pi}{T}\right);$$

Relatia dintre planele *s* si *z* in cazul invariantei raspunsului la impuls

$$s=\sigma+j\omega\;;z=re^{j\Omega}=x+jy\;;\;z=e^{sT}\Rightarrow re^{j\Omega}=e^{\sigma T}\cdot e^{j\omega T}\Leftrightarrow\begin{cases}r=e^{\sigma T}\\ \omega T=\Omega+2k\pi\end{cases}.$$

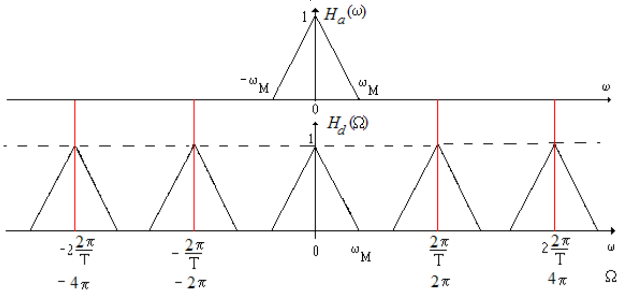


Pentru ca sa nu apara suprapuneri de tip "alias" in raspunsul in frecventa al sistemului digital echivalent, este necesar ca raspunsul in frecventa al sistemului analogic sa fie cuprins in intregime in banda de frecvente de la $-\pi/T$ la π/T. Cu alte cuvinte sistemul analogic trebuie sa fie de banda limitata la $\omega_M\le\pi/T$ si sa se esantioneze cu $\omega_e=\frac{2\pi}{T}$.

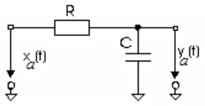
Relatia intre raspunsurile in frecventa ale sistemelor echivalente in cazul invariantei raspunsului la impuls

$$H_d(z)\Big|_{z=e^{sT}}=\sum_{k=-\infty}^{\infty}H_a\left(s-j\frac{2k\pi}{T}\right);\ z=e^{j\Omega};\ s=j\omega\Rightarrow$$

$$H_d\left(e^{j\Omega}\right)=\sum_{k=-\infty}^{\infty}H_a\left(\omega-k\frac{2\pi}{T}\right)\Big|_{\omega=\frac{\Omega}{T}}=\sum_{k=-\infty}^{\infty}H_a\left(\frac{\Omega-k2\pi}{T}\right).$$



Exemplu

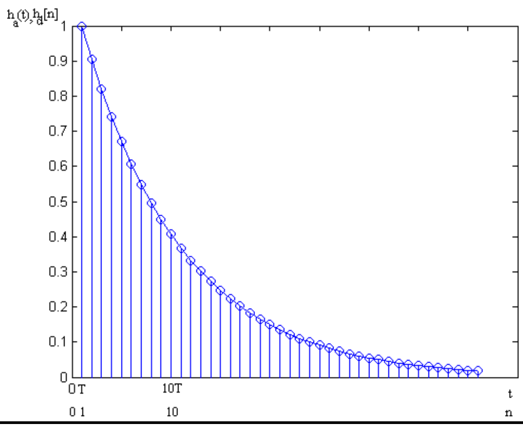


$$RC\frac{dy_a(t)}{dt}+y_a(t)=x_a(t)$$

$$H_a(s)=\frac{1}{1+\frac{s}{\omega_0}};\omega_0=\frac{1}{\tau}=\frac{1}{RC}.\ H_a(\omega)=\frac{1}{1+j\frac{\omega}{\omega_0}}=\frac{\omega_0}{\omega_0+j\omega}.$$

1. Raspunsurile la impuls

$$h_a(t)=\omega_0e^{-\omega_0t}\sigma(t)=\frac{1}{\tau}e^{-\frac{t}{\tau}}\sigma(t).\ h_d[n]=Th_a(nT)=\frac{T}{\tau}e^{-\frac{nT}{\tau}}\sigma[n].$$



$T=1,\tau=1$

2. Raspunsurile indiciale

$$Y_a(s)=H_a(s)X_a(s)=\frac{\omega_0}{(\omega_0+s)s}=\frac{-1}{\omega_0+s}+\frac{1}{s}\leftrightarrow\left(1-e^{-\frac{t}{\tau}}\right)\sigma(t),$$

$$Y_d(z)=H_d(z)X_d(z)=\frac{\frac{T}{\tau}}{\left(1-e^{-\frac{T}{\tau}}z^{-1}\right)\left(1-z^{-1}\right)}=$$

$$=-\frac{T}{\tau}\cdot\frac{e^{-\frac{T}{\tau}}}{1-e^{-\frac{T}{\tau}}}\cdot\frac{1}{1-e^{-\frac{T}{\tau}}z^{-1}}+\frac{T}{\tau}\cdot\frac{1}{1-e^{-\frac{T}{\tau}}}\cdot\frac{1}{1-z^{-1}}\leftrightarrow$$

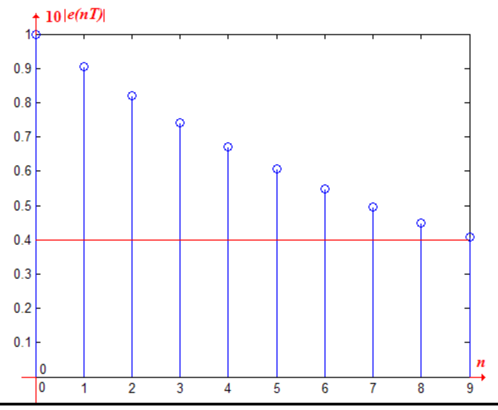
$$\leftrightarrow\frac{T}{\tau}\cdot\frac{1}{1-e^{-\frac{T}{\tau}}}\left(1-e^{-\frac{T}{\tau}}\cdot e^{-\frac{nT}{\tau}}\right)\sigma[n].$$

$$y_a(t)=\left(1-e^{-\frac{t}{\tau}}\right)\sigma(t);\ y_d[n]=\frac{T}{\tau}\cdot\frac{1}{1-e^{-\frac{T}{\tau}}}\left(1-e^{-\frac{T}{\tau}}\cdot e^{-\frac{nT}{\tau}}\right)\sigma[n].$$

Apare eroarea: $e(nT)=y_a(nT)-y_d(nT)$. Consideram ca $\frac{T}{\tau}<<1$, de unde $e^{-\frac{T}{\tau}}\cong 1-\frac{T}{\tau}$.

$y_a(nT)=1-e^{-\frac{nT}{\tau}}, n\geq 0$ si $y_d(nT)\cong 1-\left(1-\frac{T}{\tau}\right)e^{-\frac{nT}{\tau}}, n\geq 0$ si $\frac{T}{\tau}<<1$. In final:

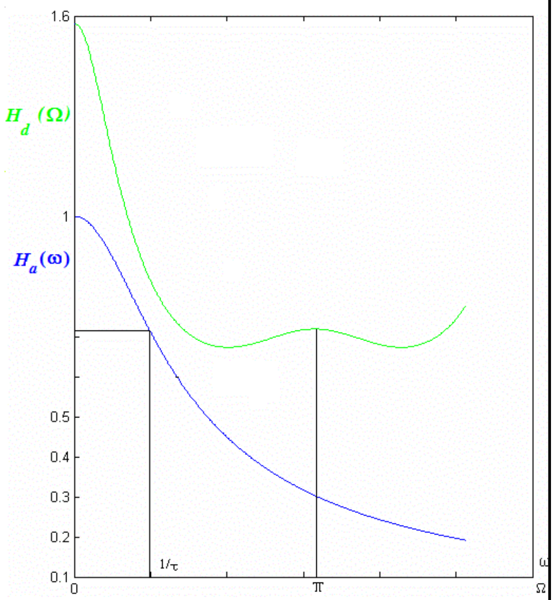
$e(nT)\cong-\frac{T}{\tau}e^{-\frac{nT}{\tau}}, \frac{T}{\tau}<<1, n\geq 0$. Pentru $\frac{T}{\tau}=\frac{1}{10}, e^{-\frac{T}{\tau}}\cong 0{,}905, |e(nT)|\cong 0{,}1e^{-\frac{n}{10}}$.



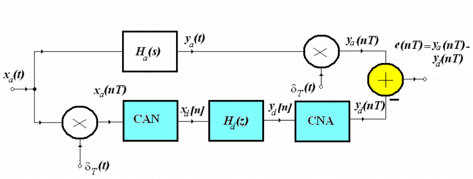
3. Raspunsurile in frecventa

$H_a(\omega)=\frac{1}{\frac{1}{\tau}+j\omega}$ si $H_d(\Omega)=\frac{1}{1-e^{-\frac{T}{\tau}}\cdot e^{-j\Omega}}$.

$T=1;\tau=1.$



Echivalarea sistemelor de banda limitata prin metoda invariantei raspunsului indicial



$x_a(t)=\sigma(t)\leftrightarrow X_a(s)=\frac{1}{s}$;

$y_a(t)=h_a(t)*\sigma(t)\leftrightarrow Y_a(s)=H_a(s)\cdot\frac{1}{s}$.

Consideram ca $x_d[n]=\sigma[n]\leftrightarrow X_d(z)=\frac{1}{1-z^{-1}}$.

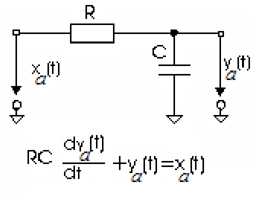
Relatia $H_d(z)=\frac{1}{X_d(z)}\mathcal{Z}\{\mathcal{L}^{-1}\{X_a(s)H_a(s)\}(nT)\}$ *devine :* $H_d(z)=(1-z^{-1})\mathcal{Z}\left\{\mathcal{L}^{-1}\left\{\frac{H_a(s)}{s}\right\}(nT)\right\}$.

Notand raspunsurile indiciale :

$s_a(t)=h_a(t)*\sigma(t)$ *si* $s_d[n]=h_d[n]*\sigma[n]$ *ultima relatie devine :*

$\mathcal{Z}^{-1}\left\{\frac{H_d(z)}{1-z^{-1}}\right\}[n]=\mathcal{L}^{-1}\left\{\frac{H_a(s)}{s}\right\}(nT)\Leftrightarrow s_d[n]=s_a(nT)$.

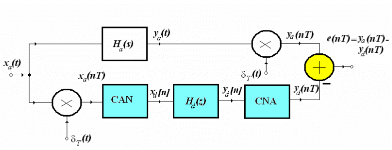
Exemplu



$$s_a(t)=\left(1-e^{-\frac{t}{\tau}}\right)\sigma(t)\Rightarrow s_d[n]=\left(1-e^{-\frac{nT}{\tau}}\right)\sigma[n]\Rightarrow$$

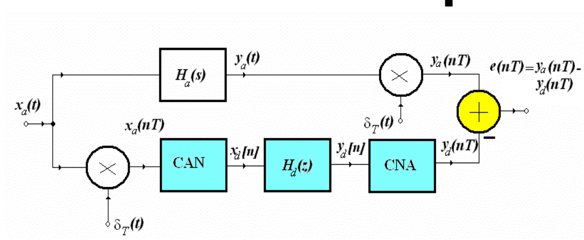
$$\Rightarrow h_d[n]=\left(e^{\frac{T}{\tau}}-1\right)\left(e^{-\frac{nT}{\tau}}\sigma[n]-\delta[n]\right)\neq h_a(nT).$$

Pentru $\frac{T}{\tau}<<1$ avem: $h_d[n]\cong\frac{T}{\tau}e^{-\frac{nT}{\tau}}\sigma[n]-\frac{T}{\tau}\delta[n]$.



Eroarea de echivalare care apare daca aplicam la intrarile celor doua sisteme impulsurile unitare $\delta(t)$ si $\delta[n]$ este: $e(nT)=h_a(nT)-h_d[n]\cong\frac{T}{\tau}\delta[n],\ \frac{T}{\tau}<<1.$

Echivalarea sistemelor de banda limitata prin metoda invariantei la semnal rampa

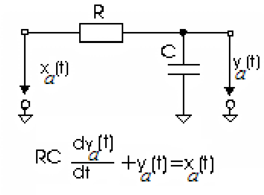


$$x_a(t)=t\sigma(t)\Rightarrow x_d[n]=n\sigma[n];$$

$$X_a(s)=\frac{1}{s^2};\ X_d(z)=\frac{z^{-1}}{\left(1-z^{-1}\right)^2}.$$

$$H_d(z)=\frac{\left(1-z^{-1}\right)^2}{z^{-1}}\mathcal{Z}\left\{\mathcal{L}^{-1}\left\{\frac{H_a(s)}{s^2}\right\}(nT)\right\}(z).$$

Exemplu



$$\frac{H_a(s)}{s^2} = \frac{\omega_0}{s^2(\omega_0+s)} = \frac{\frac{1}{\omega_0}}{\omega_0+s} - \frac{\frac{1}{\omega_0}}{s} + \frac{1}{s^2} \leftrightarrow \left[t - \frac{1}{\omega_0}\left(1-e^{-\omega_0 t}\right)\right]\sigma(t) = y_a(t);$$

$$y_a(nT) = \left[nT - \frac{1}{\omega_0}\left(1-e^{-n\omega_0 t}\right)\right]\sigma[n] \Rightarrow y_d[n] = \left[nT - \frac{1}{\omega_0}\left(1-e^{-n\omega_0 T}\right)\right]\sigma[n].$$

$$H_d(z) = \frac{\left(1-z^{-1}\right)^2}{z^{-1}} Z\left\{\left[nT - \frac{1}{\omega_0}\left(1-e^{-n\omega_0 t}\right)\right]\sigma[n]\right\}.$$

Pentru a calcula aceasta transformata tinem seama de relatiile:

$$Tn\sigma[n] \leftrightarrow T\frac{z^{-1}}{\left(1-z^{-1}\right)^2};\ -\frac{1}{\omega_0}\sigma[n] \leftrightarrow -\frac{1}{\omega_0}\frac{1}{1-z^{-1}};\ \frac{1}{\omega_0}e^{-n\omega_0 T}\sigma[n] \leftrightarrow \frac{1}{\omega_0}\frac{1}{1-e^{-\omega_0 T}z^{-1}}.$$

In consecinta:

$$H_d(z) = T - \frac{1}{\omega_0}\frac{1-z^{-1}}{z^{-1}} + \frac{1}{\omega_0}\frac{1-2z^{-1}+z^{-2}}{z^{-1}\left(1-e^{-\omega_0 T}z^{-1}\right)}.$$

$$H_d(z) = T - \frac{1}{\omega_0}\frac{1-z^{-1}}{z^{-1}} + \frac{1}{\omega_0}\frac{1-2z^{-1}+z^{-2}}{z^{-1}\left(1-e^{-\omega_0 T}z^{-1}\right)}.$$

Daca $\omega_0 T = \frac{T}{\tau} << 1$ atunci $e^{\omega_0 T} \cong 1 + \omega_0 T = 1 + \frac{T}{\tau}$ si

$$H_d(z) \cong T + \tau\left(1 - 1 - \frac{T}{\tau}\right) + \tau\left(1+\frac{T}{\tau}\right)\left(\frac{T}{\tau}\right)^2 \frac{1}{1-e^{-\frac{T}{\tau}}z^{-1}}$$

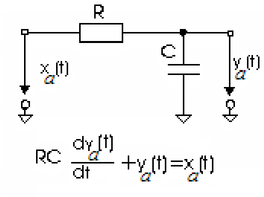
sau $H_d(z) \cong \tau\left(1+\frac{T}{\tau}\right)\left(\frac{T}{\tau}\right)^2 \frac{1}{1-e^{-\frac{T}{\tau}}z^{-1}} \Rightarrow$

$$\Rightarrow h_d[n] \cong \tau\left(1+\frac{T}{\tau}\right)\left(\frac{T}{\tau}\right)^2 e^{-\frac{T}{\tau}n}\sigma[n] \cong \tau\left(\frac{T}{\tau}\right)^2 e^{-\frac{T}{\tau}n}\sigma[n].$$

Eroarea de echivalare care apare in cazul in care la intrarile celor doua sisteme se aduc impulsuri unitare este:

$$e(nT) \cong \frac{1}{\tau}e^{-\frac{T}{\tau}n}\left(1-T^2\right)\sigma[n] \xrightarrow{n\to\infty} 0.$$

Echivalarea unui sistem analogic prin aproximarea ecuatiei diferentiale care il descrie printr-o ecuatie cu diferente finite



$$\tau = RC = \frac{1}{\omega_0} \Rightarrow \tau\frac{dy_a(t)}{dt} + y_a(t) = x_a(t) \Rightarrow H_a(s) = \frac{1}{1+s\tau}.$$

Derivata intai se poate aproxima prin:

$$\left.\frac{dy_a(t)}{dt}\right|_{t=nT} \cong \frac{y_a(nT) - y_a(nT-T)}{T} = \frac{y_d[n] - y_d[n-1]}{T} \Rightarrow$$

$$\frac{\tau}{T}\left(y_d[n] - y_d[n-1]\right) + y_d[n] = x_d[n] \text{ sau}$$

$$\left(\frac{\tau}{\mathrm{T}}+1\right)y[n] - \frac{\tau}{T}y[n-1] = x[n], \Rightarrow$$

$$\Rightarrow H_d(z) = \frac{1}{\frac{\tau}{T}+1-\frac{\tau}{T}z^{-1}} = \frac{1}{1+\tau\frac{1-z^{-1}}{T}} = \left.H_a(s)\right|_{s=\frac{1-z^{-1}}{T}}.$$

Un sistem analogic este descris de ecuatia diferentiala: $\sum_{k=0}^{N} a_k \frac{d^k y(t)}{dt^k} = \sum_{k=0}^{M} b_k \frac{d^k x(t)}{dt^k} \Rightarrow H_a(s) = \frac{\sum_{k=0}^{M} b_k s^k}{\sum_{k=0}^{N} a_k s^k}$.

Am utilizat deja aproximarea: $\left.\frac{du(t)}{dt}\right|_{t=nT} \cong \frac{u[n]-u[n-1]}{T}$. Aceasta aproximare reprezinta un caz particular (obtinut pentru $k=1$) al relatiei: $\left.\frac{d^k u(t)}{dt^k}\right|_{t=nT} \cong \frac{1}{T^k}\sum_{p=0}^{k}(-1)^p C_k^p u[n-p]$. Transformata z a membrului drept este: $\frac{1}{T^k}\sum_{p=0}^{k}(-1)^p C_k^p \left(z^{-1}\right)^k \cdot U(z) = \left(\frac{1-z^{-1}}{T}\right)^k U(z)$. Substituind aproximarile derivatelor in ecuatia diferentiala se obtine: $\sum_{k=0}^{N} a_k \frac{1}{T^k}\sum_{p=0}^{k}(-1)^p C_k^p y[n-p] = \sum_{k=0}^{M} b_k \frac{1}{T^k}\sum_{p=0}^{k}(-1)^p C_k^p x[n-p]$.

Luand in ambii membri transformata Z rezulta: $\sum_{k=0}^{N} a_k \left(\frac{1-z^{-1}}{T}\right)^k Y(z) = \sum_{k=0}^{M} b_k \left(\frac{1-z^{-1}}{T}\right)^k X(z) \Rightarrow$

$H_d(z) = \frac{\sum_{k=0}^{M} b_k \left(\frac{1-z^{-1}}{T}\right)^k}{\sum_{k=0}^{N} a_k \left(\frac{1-z^{-1}}{T}\right)^k}$. Deci: $H_d(z) = \left.H_a(s)\right|_{s=\frac{1-z^{-1}}{T}}$.

Relatia dintre planele *s* si *z* in cazul aproximarii ecuatiei diferentiale printr-o ecuatie cu diferente finite

$z = x + jy = re^{j\Omega}$ si $s = \sigma + j\omega$. S-a demonstrat relatia : $s = \frac{1 - z^{-1}}{T} \Rightarrow z = \frac{1}{1 - sT}$ adica :

$r = |z| = \frac{1}{\sqrt{(1 - \sigma T)^2 + (\omega T)^2}}$. Daca $\sigma < 0 \Rightarrow 1 - \sigma T > 1 \Rightarrow (1 - \sigma T)^2 + (\omega T)^2 > 1 \Rightarrow r < 1$.

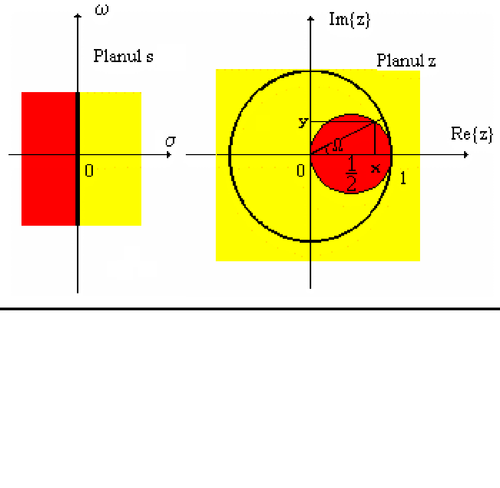
Semiplanul stang din planul s se transforma in interiorul cercului unitate din planul z.
Daca sistemul analogic este stabil si sistemul numeric echivalent va fi stabil.

Daca $\sigma = 0 \Rightarrow s = j\omega \Rightarrow z = \frac{1}{1 - j\omega T} = \frac{1 + j\omega T}{1 + (\omega T)^2} = x + jy \Rightarrow x = \frac{1}{1 + (\omega T)^2}$; $y = \frac{\omega T}{1 + (\omega T)^2}$

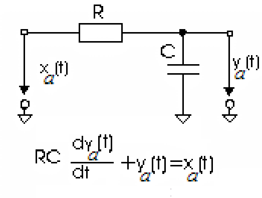
$\Rightarrow \frac{y}{x} = \omega T \Rightarrow x = \frac{1}{1 + \frac{y^2}{x^2}} \Leftrightarrow x = \frac{x^2}{x^2 + y^2} \Leftrightarrow x^2 + y^2 - x = 0 \Rightarrow \left(x - \frac{1}{2}\right)^2 + y^2 = \left(\frac{1}{2}\right)^2$.

Axa imaginara din planul s se transforma in conturul cercului de centru $(1/2, 0)$ si de raza 1/2 din planul z.

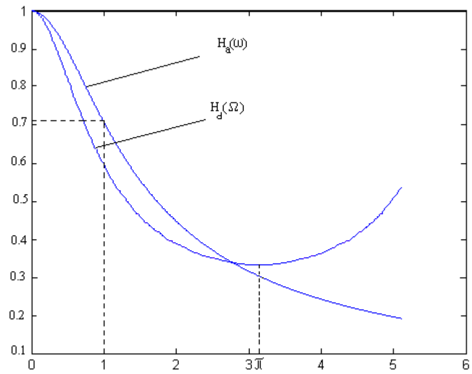
$\left(x - \frac{1}{2}\right)^2 + y^2 = \left(\frac{1}{2}\right)^2$. $\frac{y}{x} = tg\Omega = \omega T \Rightarrow \Omega = arctg\omega T$. Pentru ca sistemul numeric echivalent sa aiba raspuns in frecventa ar fi necesar ca axa imaginara din planul s sa se transforme in cercul unitate din planul z. Nu este cazul acestei metode de echivalare. Pentru $|\Omega| \le \frac{\pi}{36}$ cele 2 cercuri sunt foarte apropiate. Daca se noteaza cu ω_M frecventa maxima din raspunsul in frecventa $H_a(\omega)$ atunci conditia de mai sus se scrie : $\omega_M T = tg\Omega_M \cong \Omega_M \le \frac{\pi}{36}$. Frecventa de esantionare ω_e trebuie aleasa a. i. $\omega_e \ge 36 \cdot 2\omega_M$. Aceasta valoare este foarte mare.



Exemplu

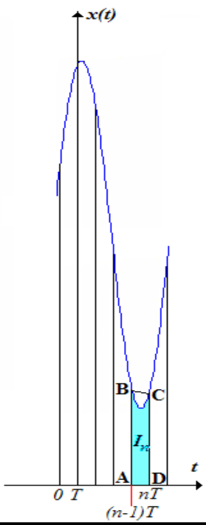


$$\tau = RC = \frac{1}{\omega_0} \Rightarrow H_a(s) = \frac{1}{1+s\tau} \Rightarrow H_d(z) = \frac{\frac{T}{T+\tau}}{1-\frac{\tau}{T+\tau}z^{-1}}$$



$\tau = 1$

Echivalarea bazata pe transformarea biliniara



Consideram cazul integratorului analogic. Acesta este descris de ecuatia diferentiala :

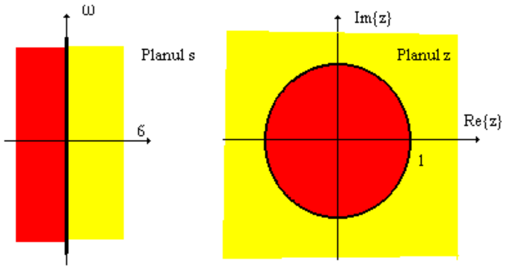
$\frac{dy}{dt} = x(t) \Rightarrow y(t) = \int_{-\infty}^{t} x(\tau)d\tau \Rightarrow H_a(s) = \frac{1}{s}$. Integrala poate fi calculata numeric folosind, de exemplu metoda trapezelor. Aria I_n poate fi aproximata cu aria trapezului ABCD.

$$I_n = y(nT) - y((n-1)T) = \int_{(n-1)T}^{nT} x(\tau)d\tau \cong \frac{(AB+CD)AD}{2} = \frac{[x(nT)+x((n-1)T)]T}{2}.$$

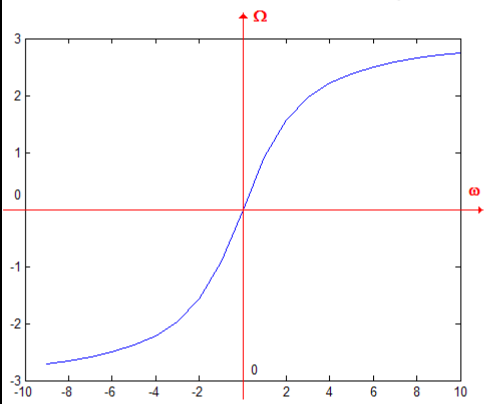
S-a obtinut : $y[n]-y[n-1] = \frac{T}{2}(x[n]+x[n-1])$. Aceasta este ecuatia cu diferente finite care caracterizeaza sistemul digital cu functia de transfer $H_d(z) = \frac{T}{2}\frac{1+z^{-1}}{1-z^{-1}} = H_a(s)\Big|_{s=\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}}$.

Relatia dintre planele *s* si *z* in cazul echivalarii pe baza transformarii biliniare

$$s = \frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}} \Leftrightarrow z = \frac{1+\frac{T}{2}s}{1-\frac{T}{2}s} \Rightarrow |z| = \sqrt{\frac{\left(1+\frac{T}{2}\sigma\right)^2+\left(\frac{T}{2}\omega\right)^2}{\left(1-\frac{T}{2}\sigma\right)^2+\left(\frac{T}{2}\omega\right)^2}} \Rightarrow \begin{cases} \sigma<0 & \Rightarrow & |z|<1 \\ \sigma=0 & \Rightarrow & |z|=1 \\ \sigma>0 & \Rightarrow & |z|>1 \end{cases}$$



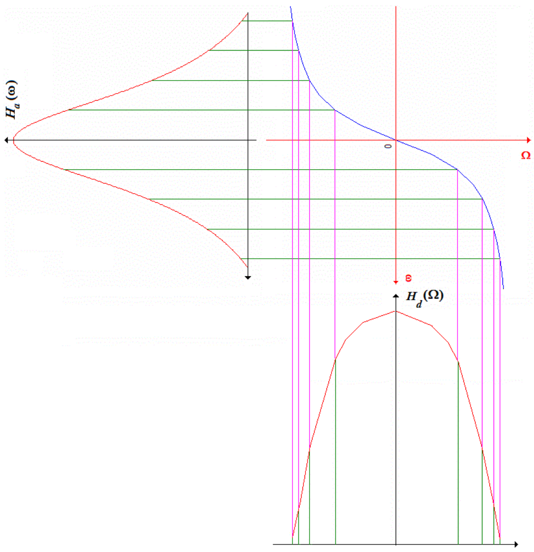
Relatia intre raspunsurile in frecventa ale sistemelor echivalente in cazul transformarii biliniare



$$s = j\omega \Rightarrow z = \frac{1+j\frac{\omega T}{2}}{1-j\frac{\omega T}{2}} = 1\cdot e^{j2arctg\frac{\omega T}{2}} = re^{j\Omega}.$$

$|z| = 1$ si $\Omega = 2arctg\frac{\omega T}{2}$.

Axa imaginara a planului *s* se transforma in cercul unitar din planul *z*. Legatura dintre frecvente este: $\omega = \frac{2}{T}tg\frac{\Omega}{2}$; $\Omega = 2arctg\frac{\omega T}{2}$.



Raspunsul in frecventa al sistemului numeric echivalent este distorsionat datorita legaturii neliniare dintre frecvente.